

# Staged Sums of Products

Haskell Symposium 2020

Matthew Pickering, University of Bristol

Andres Löh, Well-Typed LLP

Nicolas Wu, Imperial College London

2020-08-28

## Example: semigroup append

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class Semigroup a where  
  (<>) :: a -> a -> a -- supposed to be associative
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```

Same idea applies to product types in general ...

- ▶ Types are represented as n-ary **sums** (  $NS$  ) and n-ary **products** (  $NP$  ).
- ▶ Conversion functions and lots of combinators.
- ▶ Generic functions written in a type-safe and concise style.

WGP 2014

## True Sums of Products

Edsko de Vries  
Well-Typed LLP  
edsko@well-typed.com

Andres Löh  
Well-Typed LLP  
andres@well-typed.com

**Abstract**  
We introduce the *sum-of-products* (SOP) view for datatype-generic programming (in Haskell). While many of the libraries that are commonly in use today represent datatypes as arbitrary combinations of binary sums and products, SOP reflects the structure of datatypes more faithfully: each datatype is a single  $n$ -ary sum, where each component of the sum is a single  $n$ -ary product. This representation turns out to be expressible accurately in GHC with `data`'s extensions. The resulting list-like structure of datatypes allows for a definition of powerful high-level traversal combinators in the form of generic functions in the `generics-sop` library. The definition of `g` of the SOP view is

ist, even within a single programming language such as Haskell. These approaches differ in a multitude of different ways, such as which and how many functions are predefined, which features of the Haskell language are being used, how portable they are, how much emphasis on efficiency they place, and so on. Their main distinguishing feature, however, is how they view the structure of datatypes.

Not all of these views are completely different from each other. Many libraries are based on variations of what is typically called a "sum of products" view. For example, the generic representation of a binary tree type such as

```
data Tree a = Leaf | Branch (Tree a) a (Tree a)
```

the `GHC.Generics` library is essentially isomorphic to

## Generic semigroup append

```
gsappend :: (IsProductType a xs, All Semigroup xs) => a -> a -> a
gsappend a1 a2 =
  productTypeTo
    (czipWithNP (Proxy @Semigroup) (mapIII (<>))
     (productTypeFrom a1) (productTypeFrom a2))
```

## Generic semigroup append

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sappendFoo :: Foo -> Foo -> Foo
sappendFoo = gsappend
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productTypeFrom (Foo is o t) = I is :* I o :* I t :* Nil
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productTypeTo
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```

`mapIII :: (a -> b -> c) -> I a -> I b -> I c`  
`mapIII op (I x) (I y) = I (op x y)`

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```
Foo [1, 2, 3, 4] LT "haskell"
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# What about efficiency?

hand-written



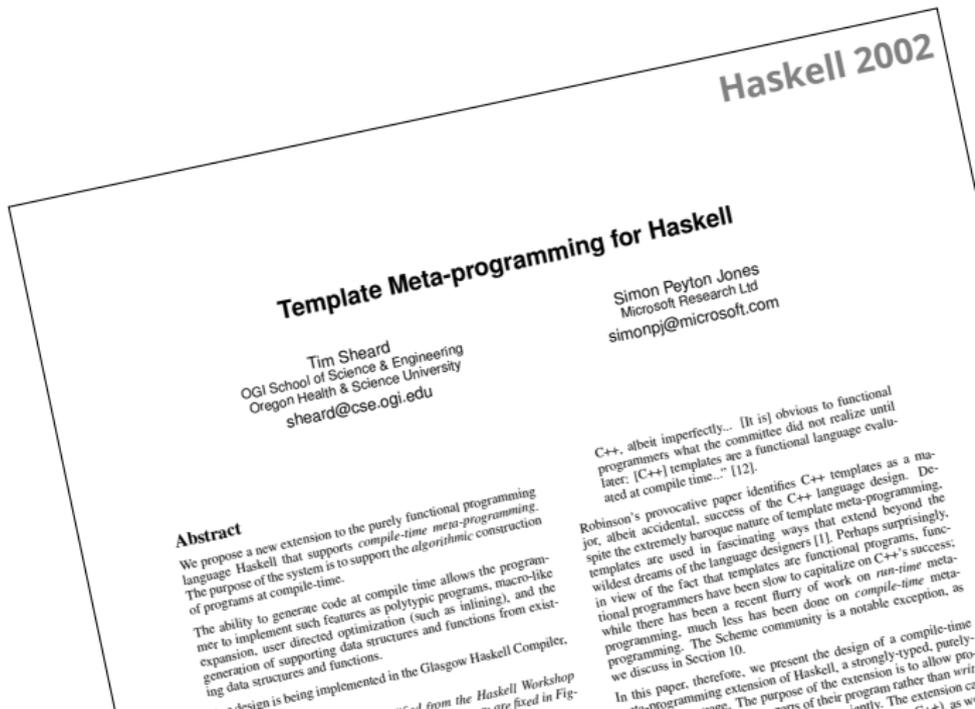
1

## What about efficiency?



# Typed Template Haskell

- ▶ A **typed subset** of Template Haskell.
- ▶ Construct and use Haskell **expressions** at **compilation time**.



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Haskell 2002

Meta-programming for Haskell

Simon Peyton Jones  
Microsoft Research Ltd  
simonpj@microsoft.com

MetaML: Multi-Stage Programming with Explicit Annotations  
Wald Taha & Tom Shard  
Oregon Graduate Institute of Science and Technology  
{waldtaha, shard}@ospi.ogi.edu

PEPM 1997

## Abstract

We introduce MetaML, a practical, non-strictly typed multi-stage programming language. MetaML allows the programmer to construct, compile, and execute code fragments in a type-safe manner. Code fragments can contain state-escaping pointers, but we ensure that the language always terminates. MetaML performs type-checking at compile-time and for all before the execution of the first code fragment. This is necessary to ensure that the program never generates pointers to memory that has already been freed. A thesis of this paper is that multi-stage programming is a natural way to express stateful computations in a typed language. We show that this is possible, and that it is a natural way to express stateful computations in a typed language.

## 1.1 Multi-Stage Programs and Languages

The concept of a stage arises naturally in a wide variety of situations. For a compiled language, the execution of a program involves two distinct stages: compilation and execution. These distinct stages appear at the compiler, and the programmer's code; they are separated by the compiler.

Workshop  
Fixed in Fig.

C++, albeit imperfectly... It is obvious to functional programmers what the committee did not realize until later: [C++] templates are a functional language evaluated at compile time..." [12].

Robinson's provocative paper identifies C++ templates as a major, albeit accidental, success of the C++ language design. Despite the extremely baroque nature of template meta-programming, templates are used in fascinating ways that extend beyond the wildest dreams of the language designers [1]. Perhaps surprisingly, functional programmers have been slow to capitalize on C++'s success; while there has been a recent flurry of work on *run-time* meta-programming, much less has been done on *compile-time* meta-programming. The Scheme community is a notable exception, as we discuss in Section 10.

In this paper, therefore, we present the design of a compile-time, purely functional extension of Haskell, a strongly-typed, purely functional programming language. The purpose of the extension is to allow programmers to write parts of their program rather than write parts of their program at compile-time. The extension can be used as a library, or as a pre-processor, or as a compiler extension.

# Typed Template Haskell

- ▶ A **typed subset** of Template Haskell.
- ▶ Construct and use Haskell **expressions** at **compilation time**.

Haskell 2002

Meta-programming for Haskell

Simon Peyton Jones  
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PEPM 1997

MetaML: Multi-Stage Programming with Explicit Annotations  
Wald Taha & Tom Shard  
Oregon Graduate Institute of Science and Technology  
{waha@cs.oregonstate.edu, shard@cs.oregonstate.edu}

-- functional

The Internet, 2013

Geoffrey Mainland Home CV Publications Schedule

## Type-Safe Runtime Code Generation with (Typed) Template Haskell

31 May 2013

Over the past several weeks I have implemented most of Simon Peyton Jones' [proposal for a major revision to Template Haskell](#). This brings several new features to Template Haskell, including:

1. Typed Template Haskell brackets and splices.
2. Pattern splices and local declaration splices.
3. The ability to add (and use) new top-level declarations from within top-level splices.

The code in this post is available on [GitHub](#). The GHC wiki

# Staging constructs

## Quotes

$$\frac{e :: t}{[|e|] :: \text{Code } t}$$

Prevent reduction, build an AST.

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Prevent reduction, build an AST.

`type Code a = Q (TExp a)`

# Staging constructs

## Quotes

$$\frac{e :: t}{[|e|] :: \text{Code } t}$$

Prevent reduction, build an AST.

## Splices

$$\frac{e :: \text{Code } t}{\$\$e :: t}$$

Re-enable reduction, insert into an AST.

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Re-enable reduction, insert into an AST.

**Top-level splices** insert into the current module.

# Staging constructs

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$$\frac{e :: t}{[|e|] :: \text{Code } t}$$

Prevent reduction, build an AST.

## Splices

$$\frac{e :: \text{Code } t}{\$\$e :: t}$$

Re-enable reduction, insert into an AST.

**Top-level splices** insert into the current module.

Splices and quotes cancel each other out:  $\$\$([|e|]) \rightsquigarrow e$ .

## Staged semigroup append

```
sgsappend :: (IsProductType a xs, All (Quoted Semigroup) xs) =>
            Code a -> Code a -> Code a
sgsappend c1 c2 =
  sproductTypeFrom c1 $ \ a1 -> sproductTypeFrom c2 $ \ a2 ->
  sproductTypeTo (czipWithNP (Proxy @(Quoted Semigroup))
    (mapCCC [ | (<>) | ]) a1 a2)
```

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        (mapCCC [||(<>)||]) a1 a2)
```

```
sappendFoo :: Foo -> Foo -> Foo
sappendFoo foo1 foo2 = $$ (sgsappend [||foo1||] [||foo2||])
```

## Staged semigroup append

```
sappendF00 foo1 foo2
```

## Staged semigroup append

```
$$(sgsappend [||foo1||] [||foo2||])
```

## Staged semigroup append

```
$$(  
  sproductTypeFrom [||foo1||] $ \ a1 ->  
    sproductTypeFrom [||foo2||] $ \ a2 ->  
      sproductTypeTo  
        (czipWithNP (Proxy @ (Quoted Semigroup))  
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          a1  
          a2)  
)
```

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$$(  
  sproductTypeFrom [||foo1||] $ \ a1 ->  
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      sproductTypeTo  
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          (mapCCC [||(<>)||])  
          a1  
          a2)  
)
```

```
sproductTypeFrom :: Code Foo -> (NP C '[[Int], Ordering, Text] -> Code r) -> Code r  
sproductTypeFrom foo k =  
  [||case $$foo of { Foo is o t ->  
    $$k (C [||is||] :* C [||o||] :* C [||t||] :* Nil)}  
  ||]
```

## Staged semigroup append

```
$$(  
  sproductTypeFrom [||foo1||] $ \ a1 ->  
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      sproductTypeTo  
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          (mapCCC [||(<>)||])  
          a1  
          a2)  
)
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sproductTypeFrom :: Code Foo -> (NP C '[[Int], Ordering, Text] -> Code r) -> Code r  
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```

```
  [||case $$foo of { Foo is o t ->  
    $$k (C [||is||] :* C [||o||] :* C [||t||] :* Nil)}  
  ||]
```

```
newtype C a = C (Code a)
```

## Staged semigroup append

```
$$(  
  [| case foo1 of { Foo is1 o1 t1 ->  
    $$ (sproductTypeFrom [|foo2|] $ \ a2 ->  
      sproductTypeTo  
        (czipWithNP (Proxy @(Quoted Semigroup))  
          (mapCCC [|(<>)|])  
          (C [|is1|] :* C [|o1|] :* C [|t1|] :* Nil)  
          a2)) }  
  |]  
)
```

## Staged semigroup append

```
$$(  
  [| case foo1 of { Foo is1 o1 t1 ->  
    case foo2 of { Foo is2 o2 t2 ->  
      $$ (sproductTypeTo  
        (czipWithNP (Proxy @(Quoted Semigroup))  
          (mapCCC [| (<>) |])  
          (C [| is1 |] :* C [| o1 |] :* C [| t1 |] :* Nil)  
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  |]  
)
```

## Staged semigroup append

```
$$(  
  [| case foo1 of { Foo is1 o1 t1 ->  
    case foo2 of { Foo is2 o2 t2 ->  
      $$ (sproductTypeTo  
        ( mapCCC [|(<>)|] (C [|is1|]) (C [|is2|])  
          :* mapCCC [|(<>)|] (C [|o1|]) (C [|o2|])  
            :* mapCCC [|(<>)|] (C [|t1|]) (C [|t2|])  
              :* Nil  
            )) } }  
  |]  
)
```

## Staged semigroup append

```
$$(  
  [|] case foo1 of { Foo is1 o1 t1 ->  
    case foo2 of { Foo is2 o2 t2 ->  
      $$ (sproductTypeTo  
        (  mapCCC [|](<>)|] (C [|]is1|]) (C [|]is2|])  
        :* mapCCC [|](<>)|] (C [|]o1|]) (C [|]o2|])  
        :* mapCCC [|](<>)|] (C [|]t1|]) (C [|]t2|])  
        :* Nil  
      )) } }  
  [|]  
)
```

`mapCCC` :: Code (a -> b -> c) -> C a -> C b -> C c  
`mapCCC op (C x) (C y) = C [|]$$$op $$$ $$$y|]`

## Staged semigroup append

```
$$(  
  [| case foo1 of { Foo is1 o1 t1 ->  
    case foo2 of { Foo is2 o2 t2 ->  
      $$ (productTypeTo  
        ( C [|(<>) is1 is2|]  
          :* C [|(<>) o1 o2|]  
            :* C [|(<>) t1 t2|]  
              :* Nil  
            )) } }  
  |]  
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## Staged semigroup append

```
$$(  
  [| case foo1 of { Foo is o t >  
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      sproductTypeTo :: NP C '[[Int], Ordering, Text] -> Code Foo  
      sproductTypeTo (C is :* C o :* C t :* Nil) = [| Foo $$is $$o $$t|]  
        $$ (sproductTypeTo  
          ( C [| (<>) is1 is2|]  
            :* C [| (<>) o1 o2|]  
            :* C [| (<>) t1 t2|]  
            :* Nil  
          )) } }  
  |]  
)
```

## Staged semigroup append

```
$(  
  [| case foo1 of { Foo is1 o1 t1 ->  
    case foo2 of { Foo is2 o2 t2 ->  
      Foo  
        ((<>) is1 is2)  
        ((<>) o1 o2)  
        ((<>) t1 t2) } }  
  |]  
)
```

## Staged semigroup append

```
case foo1 of { Foo is1 o1 t1 ->  
  case foo2 of { Foo is2 o2 t2 ->  
    Foo ((<>) is1 is2) ((<>) o1 o2) ((<>) t1 t2) } }
```

## Staged semigroup append

```
case foo1 of { Foo is1 o1 t1 ->  
  case foo2 of { Foo is2 o2 t2 ->  
    Foo ((<>) is1 is2) ((<>) o1 o2) ((<>) t1 t2) } }
```

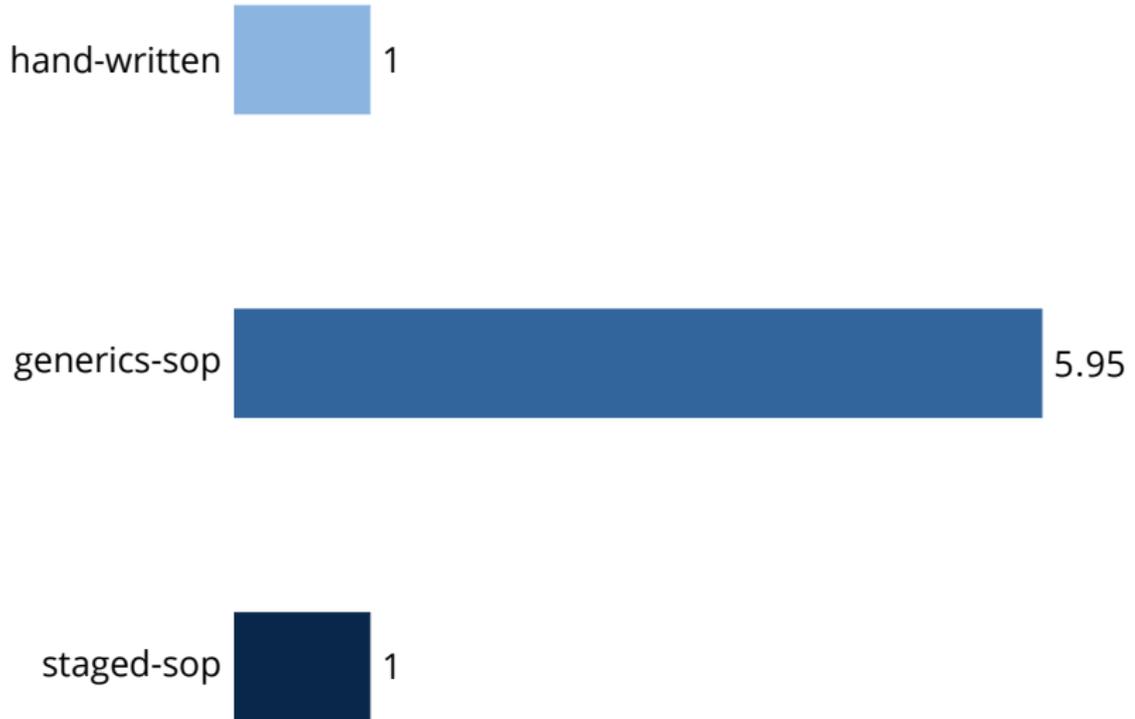
This is **obviously equivalent** to the hand-written version:

```
sappendFoo :: Foo -> Foo -> Foo  
sappendFoo (Foo is1 o1 t1) (Foo is2 o2 t2) =  
  Foo (is1 <> is2) (o1 <> o2) (t1 <> t2)
```

# What about efficiency?



# What about efficiency?



- ▶ A variant of generics-sop.
- ▶ Can reuse the `NS` and `NP` types, because they are already parameterized over a type constructor.
- ▶ Conversion functions use `C` rather than `I` as the type constructor.
- ▶ Can reuse nearly all of the provided combinators for working with sums and products.
- ▶ Requires proper handling of class constraints in Typed Template Haskell.

## Quotes and constraints

`(<>) :: Semigroup a => a -> a -> a`

---

`[| (<>) |] :: ...`

## Quotes and constraints

```
(<>) :: Semigroup a => a -> a -> a
```

---

```
[|(<>)|] :: Semigroup a => Code (a -> a -> a)
```

GHC 8.10 ad-hoc answer, which is **wrong**.

## Quotes and constraints

```
(<>) :: Semigroup a => a -> a -> a
```

---

```
[|(<>)|] :: Code (Semigroup a => a -> a -> a)
```

An option once impredicativity is available, but **not first class** (does not allow to decouple the constraint from the quote).

## Quotes and constraints

$$(\langle \rangle) :: \text{Semigroup } a \Rightarrow a \rightarrow a \rightarrow a$$

---

$$[|(\langle \rangle)|] :: \text{Quoted Semigroup } a \Rightarrow \text{Code } (a \rightarrow a \rightarrow a)$$

Our answer, which reflects that we need the constraint satisfied **when this fragment is spliced**, not when it is constructed.

## Quotes and constraints

$$(<>) :: \text{Semigroup } a \Rightarrow a \rightarrow a \rightarrow a$$

---

$$[| (<>) |] :: \text{Quoted Semigroup } a \Rightarrow \text{Code } (a \rightarrow a \rightarrow a)$$

Our answer, which reflects that we need the constraint satisfied **when this fragment is spliced**, not when it is constructed.

Implemented in a GHC branch; GHC proposal to follow.

- ▶ More examples of staged generic functions.
- ▶ A more detailed explanation of `Quoted` .
- ▶ Related work.

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## Staged Sums of Products

Andres Löh  
Well-Typed LLP  
andres@well-typed.com

Nicolas Wu  
Department of Computing  
Imperial College London  
United Kingdom  
n.wu@imperial.ac.uk

Matthew Pickering  
Department of Computer Science  
University of Bristol  
United Kingdom  
matthew.pickering@bristol.ac.uk

**Abstract**  
Generic programming libraries have historically traded efficiency in return for convenience, and the `generics-sop` library is no exception. It offers a simple, uniform, representation of all datatypes precisely as a sum of products, making it easy to write generic functions. We show how to finally make `generics-sop` fast through the use of staging with Typed Template Haskell.

**CCS Concepts:** • Software and its engineering → Functional languages.

**Keywords:** generic programming, staging

**ACM Reference Format:**  
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We can provide a `Semigroup` instance for such a type, relying on the existing `Semigroup` instances for its components. The `semigroup` operation for `Foo` can be defined as

```
sappendFoo :: Foo -> Foo -> Foo
sappendFoo (Foo !s1 o1 t1) (Foo !s2 o2 t2) =
  Foo (!s1 o !s2) (o1 o o2) (t1 o t2)
```

This is a typical generic programming pattern: we match on the sole constructor of a datatype, apply the `semigroup` operation (`o`) pointwise to its components, and apply the constructor again. None of this is specific to `Foo`; it all works whenever we have a single-constructor datatype where all components have the necessary `Semigroup` instances. Using `generics-sop`, we can therefore define

```
gsappend :: (sProductType a xs, All Semigroup xs) => a -> a -> a
gsappend a1 a2 = productTypeTo
  (czipWithop (Proxy @Semigroup) (mapM1 (o))
   (productTypeFrom a1) (productTypeFrom a2))
```

Exactly the pattern described above. The constructor `productTypeFrom` must be a single-constructor datatype, and `o` must be an instance of `Semigroup`.

# Conclusions

- ▶ We can finally write datatype-generic programs at a **high level**, with **type safety** and **reliable performance**.
- ▶ The identified improvements to constraint handling in Typed Template Haskell are independently useful.
- ▶ It is wonderful that we can reuse so much of the original generics-sop library.
- ▶ Nevertheless, staging in this style is also applicable to other generic programming approaches such as `GHC.Generics` and `SYB`.

## Try the prototype:

<https://github.com/well-typed/generics-sop/tree/staged-sop>

(README has instructions on how to build a suitably patched GHC branch.)